Numerical Simulations of Quantum Error Correction

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- We want to execute a quantum algorithm with N logical gates.
 - $N \sim 10^{12} 10^{15}$ to simulate a small molecule like Fe_2S_2 .
- Each gate is error-corrected to accuracy δ , so errors build up to
 - $N\delta$ if they add coherently (worst case, systematic bias)
 - $\sqrt{N\delta}$ if they add stochastically
- δ needs to be $\sim 1/\sqrt{N}$ to 1/N to prevent harmful error build up. • 10^{-6} to 10^{-15} for quantum chemistry (pretty vague).
- If the physical noise rate ϵ is sub threshold, then fault-tolerant error correction can produce logical gates of accuracy δ with overhead polylog($\frac{1}{\delta}$).

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Outline

- QEC simulation methods for general noise
- Problem with metrics
- 3 Channel approximations
- Decoding

- Noise modeled by some (perhaps correlated) probability distribution P(E), over $E \in \mathcal{P}^{\otimes N}$.
- To numerically simulate:
 - Sample $E \sim P(E)$ (only randomness).
 - Compute associated syndrome s(E).
 - ullet Decode: guess \hat{E} from s.
 - Check if $\hat{E} \simeq E$ (up to stabilizer).
 - Repeat to estimate logical error probability.
- For generalized noise models (think of systematic error $U^{\otimes N}$):
 - Errors are not element of the Pauli group: CPTP map
 - Syndrome is not determined by error: Born's rule.
 - Correction is not right or wrong: some fidelity.

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- Realistic noise models cannot be efficiently simulated.
 - Interacting quantum many-body problem.

Our contribution

- Tensor network methods
 - Density matrix renormalization group (DMRG).
 - Projected entangled pairs state (PEPS).
 - Multi-scale entanglement renormalization ansatz (MERA)
 - e etc.

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- Applying some noise $\mathcal E$ to $\rho = |\bar\psi\rangle\langle\bar\psi|$.
 - When \mathcal{E} is some stochastic noise, we can sample the noise instead of applying \mathcal{E} .
- Sample the syndrome bits $\operatorname{pr}_j(\pm) = \frac{1}{2}(1 \pm \operatorname{Tr}[\mathcal{E}(\rho)S_j])$.
- Decode, i.e., find a correction operation C based on the observed syndrome.
- Apply the correction to the post-measurement state ρ' .
- Evaluate the logical transformation that has been applied to the logical state.
- \bullet Repeat for different input states $\bar{\psi}$ to perform logical process tomography.
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If we can do all of this...

Simulation

INPUT

• Noise \mathcal{E} .

OUTPUT

- A syndrome s.
- The probability of that syndrome pr(s).
- The logical channel conditioned on that syndrome \mathcal{E}_s^L .

- Average channel $\overline{\mathcal{E}}_L = \sum_s \operatorname{pr}(s) \mathcal{E}_s^L$
- Average logical error $\sum_{s} \operatorname{pr}(s) \| \mathcal{E}_{s}^{L} \operatorname{id} \|$
- Error of logical average $\|\mathcal{E}_L id\|$
- etc.

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Given a physical noise rate ϵ , how much error correction do I need to achieve a logical noise rate δ ?

- How do I quantify the physical noise rate?
 - Infidelity $\epsilon = 1 \int d\psi F[\psi, \mathcal{E}(\psi)]$ has a nice statistical interpretation measured by randomized benchmarking.
 - Diamond norm \(\epsilon = \| \mathcal{E} \mathcal{I} \| \| \epsilon \) composes well, used in analytical FT studies.
 - Hilbert-Schmidt norm $\epsilon = \|\mathcal{E} \mathcal{I}\|_2$ is easy to manipulate.
 - e etc.
 - These metrics can differ significantly, e.g., ϵ vs $\sqrt{\epsilon}$.

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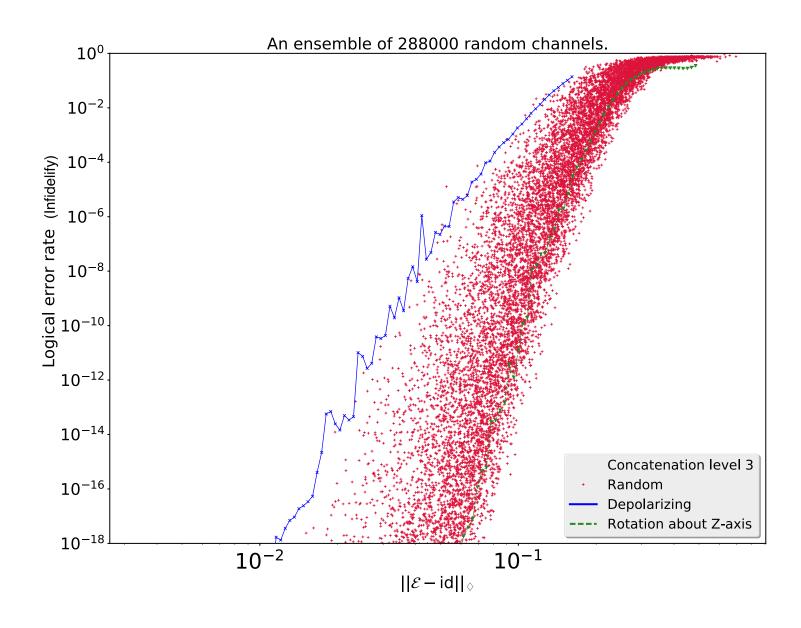
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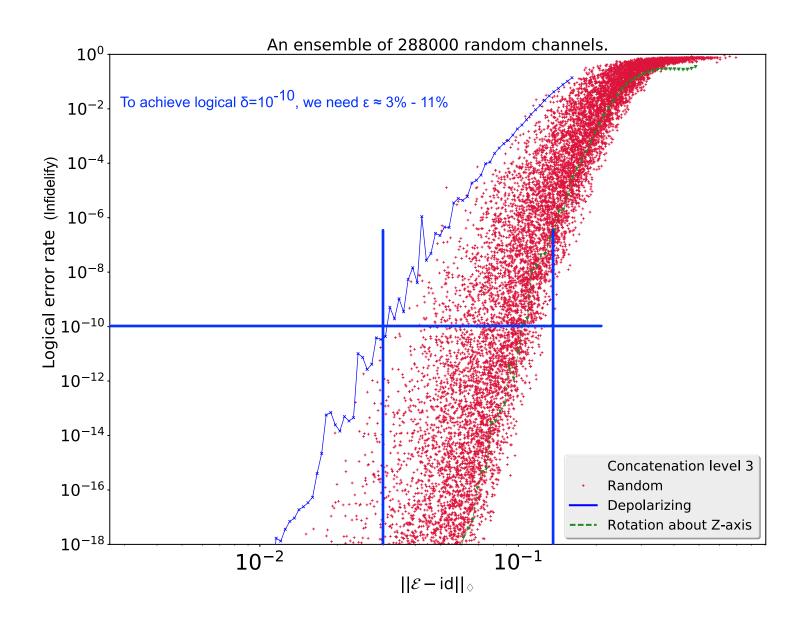
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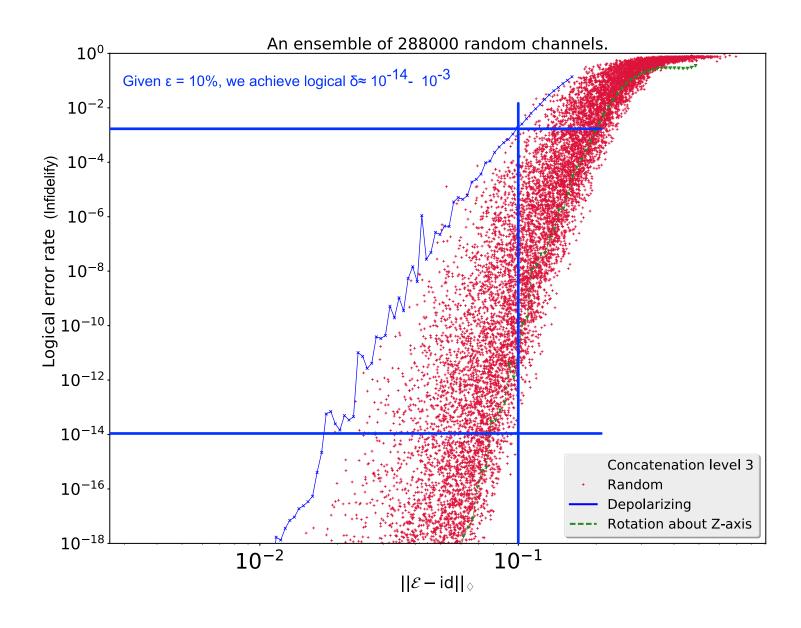
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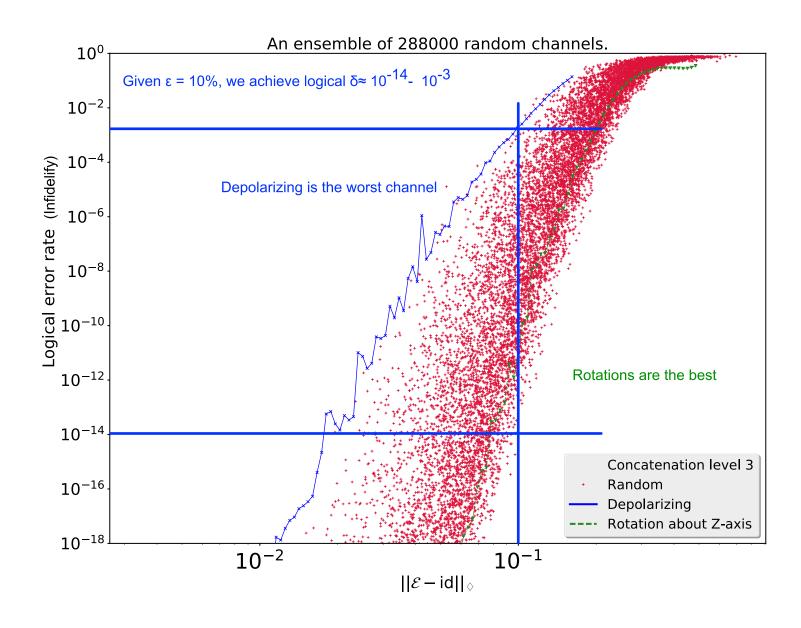
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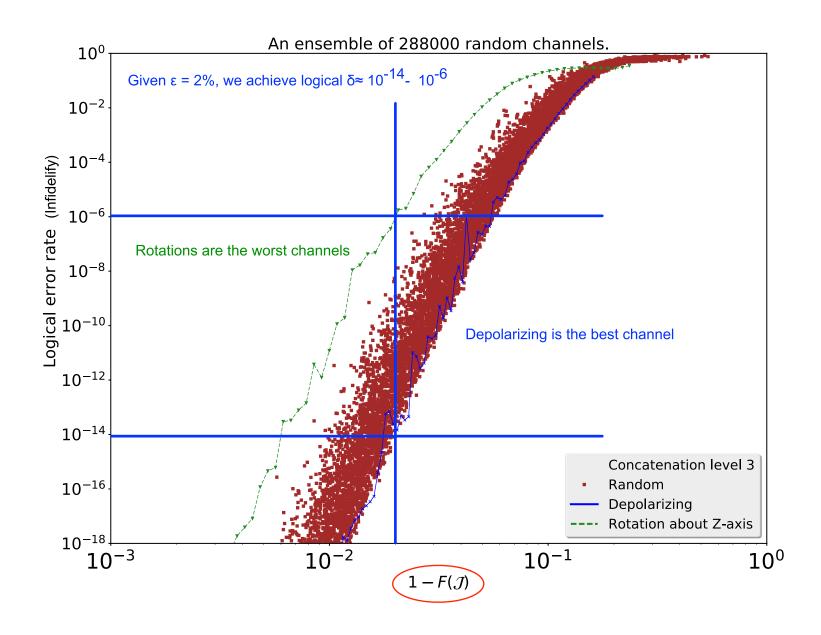
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Predictability of noise metrics

Conclusion

It is not possible to even very crudely predict the logical failure rate of a FT scheme given only the noise rate of the physical channel, as measured by any of the standard error metrics (Infidelity, Diamond norm, Channel entropy, Error probability, Euclidian norm, Trace norm).

Outline

- QEC simulation methods for general noise
- Problem with metrics
- Channel approximations
- Decoding

MC simulations are numerically very efficient, but limited to unphysical Pauli noise models.

- Let's approximate the physical channel \mathcal{E} by a Pauli channel \mathcal{P} .
 - Ignore the non-Pauli contributions to the channel
- E.g. Rotation $R_Z(\theta) = e^{i\theta Z} = \cos \theta I + i \sin \theta Z$ error

$$\rho \to (\cos \theta I + i \sin \theta Z) \rho (\cos \theta I - i \sin \theta Z)$$

is approximated by a stochastic Z error (dephazing)

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Or we could find the closest Pauli channel which is no better than
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Magasan, Puzzuoli, Granade, & Cory arXiv:1206.5407

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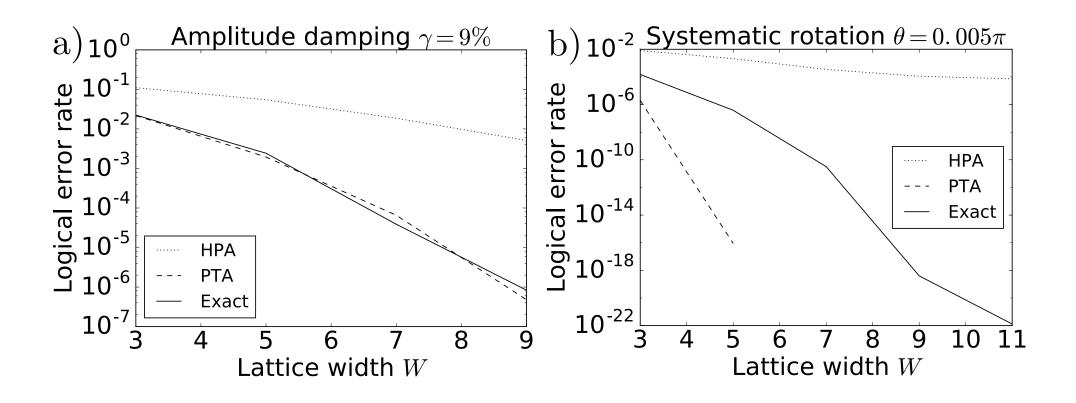
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Pauli approximations, surface code overhead



- Amplitude damping, lattice up to size $9 \times 17 = 153$ qubits.
- Depolarizing, lattice up to size $11 \times 11 = 121$ qubits.

- It is not possible to even very crudely predict the logical failure rate of a FT scheme from known Pauli approximations.
- The twirl approximation gets a good threshold estimate in the examples we looked at.
- It is essential to develop simulation methods adapted to non-Pauli noise models to get a reliable estimate of the FT overhead.

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Decoding non-Pauli noise

There are two levels of difficulty: decoding and simulating.

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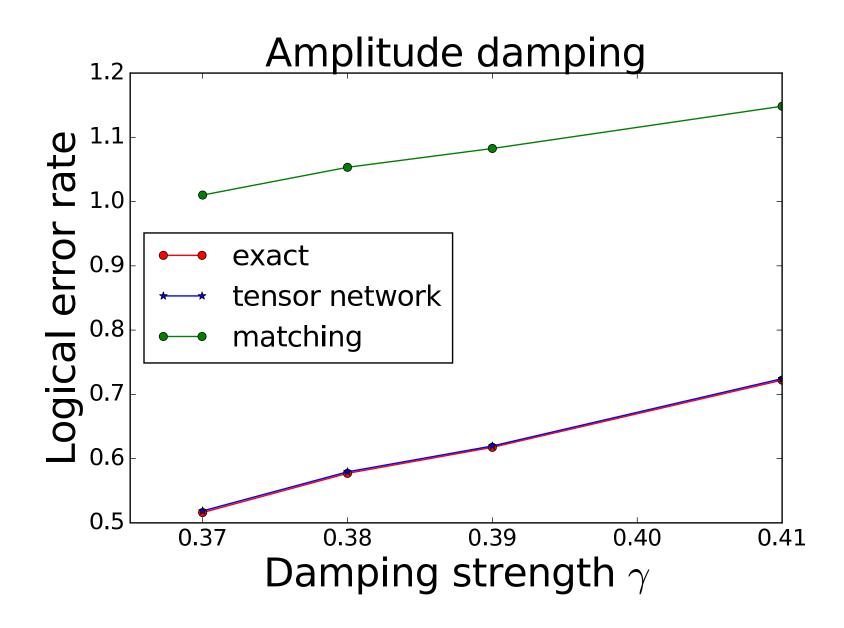
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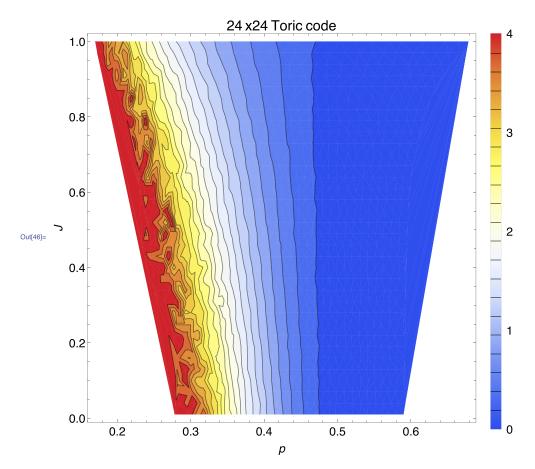
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Pauli approximations for surface code decoding



Correlated erasures on surface code



The erasure pattern is given by spin down configuration of a classical ferromagnetic Ising model in a magnetic field favoring spin ups.

Color shows

— log₁₀(logical error rate)

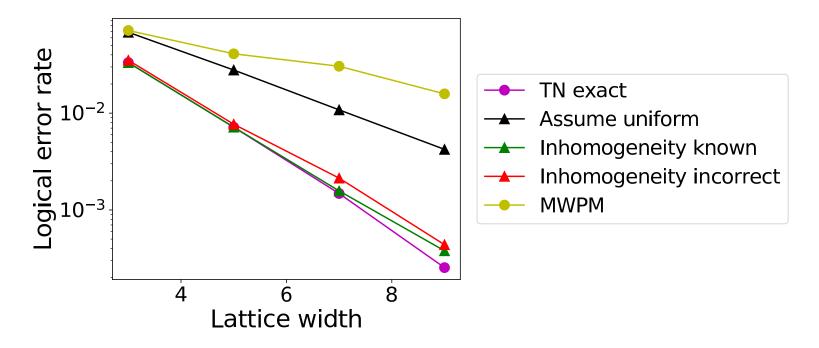
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- But for a given device, do I need to know the noise model?

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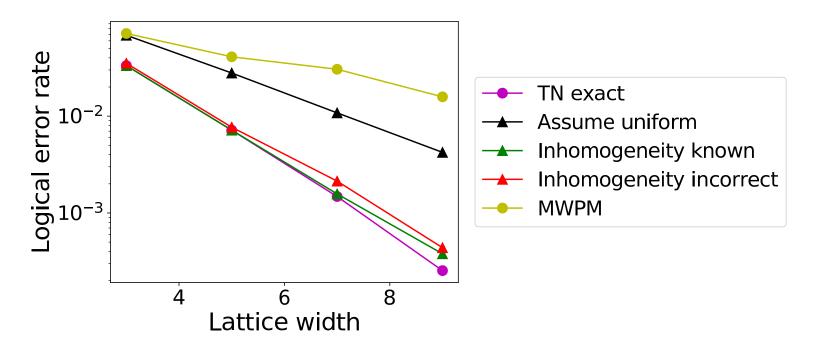
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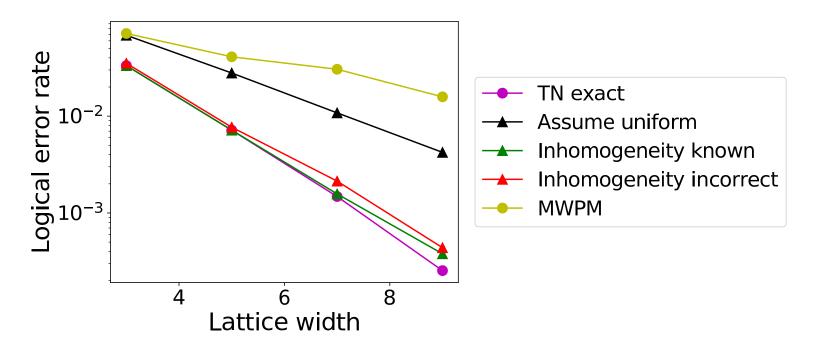
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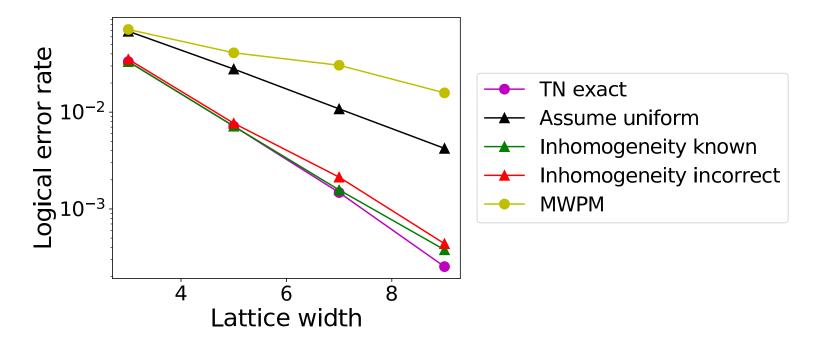
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- Reporting the quality of a qubit with a single number is not useful to predict how it will respond in a fault-tolerant scheme.

Fault-tolerance critical parameters

In a given experiment, what is the leading noise source which limits the logical accuracy?

- Developed machine learning methods to identify such critical parameters.
- Decoding non-Pauli noise with tensor networks.

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Institue Quantique @ Sherbrooke

We are looking for talented

- Graduate students
- Postdocs
- Visiting faculty/scientists

Talk to me if you have any interest.