Heilbronn Annual Conference 2018
University of Bristol
Titles and Abstracts

Martin Hairer
TBA

Sarah Zerbes
The mysteries of L-values

L-functions are one of the central objects of study in number theory. There are many beautiful theorems and many more open conjectures linking their values to all kinds of arithmetic problems. I will talk about the mysteries surrounding these L-values and describe some of the progress that has recently been made towards understanding them.

Mark Gross
Mirror symmetry in the tropics

Mirror symmetry is a geometric phenomenon in algebraic and differential geometry which has as its origins string theory of the late 1980s. Since then it has burgeoned into a sprawling field of its own. I will give a quick introduction to the subject, followed by a very explicit example of how the new field of tropical geometry gives insight into mirror symmetry.

Shekhar Khare
Automorphy of S_3, S_4 and S_5 representations over CM fields

Serre conjectured that odd, irreducible representations of the absolute Galois group G_Q of the rationals Q with image in GL_2(F_p) arise from new forms via a classical construction of Eichler and Shimura. I joint work with J-P. Wintenberger we proved this conjecture almost a decade ago. Serre's conjecture has natural generalizations to number fields and reductive groups G beyond GL_2. These remain widely open. My talk will be about taking some modest steps to address these generalized Serre-type conjectures.

Our proof of Serre's conjecture built on the work of Wiles on modularity of elliptic curves over Q. One of the first steps in his proof was to show Serre's conjecture for representations with projective image S_4. His proof for this first step uses the Langlands-Tunnell theorem which proved Artin's conjecture for complex projective representations with image S_4. The use of Langlands-Tunnell becomes problematic when we replace the field Q with say Q(i). I will talk about joint work with Patrick Allen and Jack Thorne which finds a new method to circumvent the use of the Langlands-Tunnell theorem in this context. This seems an essential step to try and extend modularity of elliptic curves over the rationals to modularity of elliptic curves over the Gaussian numbers Q(i).

Jacob Fox
Arithmetic patterns, games, and the quest for fast algorithms

In this talk, I will discuss recent advances on three seemingly disparate questions and how they relate to each other.

1. What patterns can we find in prime numbers?
2. How many cards do we need in the popular card game SET® to guarantee a valid set?
3. Can we find faster algorithms to better analyze large networks?

Finding patterns like arithmetic progressions in prime numbers has fascinated mathematicians for many centuries. More recently, people have enjoyed playing the card game SET®, and natural questions that arise from this game have been shown to be closely related to longstanding open problems in mathematics and computer science. Over the last few decades, as we strive to better understand the world through large networks, analyzing enormous data sets has become a priority. Traditional algorithms are insufficient for these purposes, and the need for faster algorithms has become apparent. Advances (some quite surprising) on these questions have used tools from a variety of areas of mathematics, including combinatorics, analysis, algebra, probability, geometry, and number theory. No prior knowledge is assumed.

Francis Brown
Single-valued integration

Single-valued functions are ubiquitous in mathematics and physics, since a well-defined problem has a well-defined answer. On the other hand, the solution to such a problem is often given by an integral, which is usually a multi-valued function of its parameters. The reason is that integration is a pairing between differential forms and chains of integration, and the latter are ambiguously defined.

In this talk, which is joint work with Clément Dupont, I will describe a way to pair differential forms with `duals of differential forms'. This defines a theory of integration which satisfies the usual rules, but is automatically single-valued. Many well-known constructions in mathematics and physics are examples of such objects. If time permits, I will explain how these ideas lead to a new theory of modular forms.